

Friday the 13th
and the
Mathematics of the Gregorian Calendar

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“Computus is the science of observing times by the movements of the sun and moon, and comparing these with each other.”

John of Sacrobosco, University of Paris, 1235 AD

Measuring the passage of time has challenged and frustrated human civilizations for thousands of years. Time is a cultural construct and different cultures will interpret the passage of time in different ways. Many traditional cultures estimated a distance traveled by the number of “sleeps” it took to get there. The passage of the seasons was often considered in terms of the moon, with each lunar cycle named for a particular event in nature that occurred at that time. In some cultures, the years were measured by the number of “snows” or winters that had passed, while some tropical cultures reckoned time by the rainy seasons, or the return of certain migratory animals.

To establish an accurate calendar, a precise determination of the length of a year is necessary. A number of different methods were developed in order to more accurately measure the length of the year. Ancient astronomers from a number of different cultures used the length of the shadow of a long vertical pole, or “gnomon,” to determine the number of days between two successive summer solstices. In the tropics, it was possible to use the passage of the sun directly overhead as a reference point. Many civilizations would use the positions of the rising and setting sun on the horizon to determine the length of a year and occurrence of the solstices.

The mathematics of our current calendar has been studied from a number of different perspectives; most recent articles have focused on the application of continued

fractions to the problem of establishing leap years. Other articles have focused on the fact that the 13th of a month is most likely to fall on a Friday (see papers by Rickey and Rasof). This paper will focus on an analysis of the frequency of Friday the 13ths within each calendar year and the insight this yields into the longer cycles that occur within our Gregorian calendar. It will also provide historical background information on the development of the Gregorian calendar and the difficulties inherent in establishing any calendar.

“What’s so bad about Friday the 13th?”

Friday the 13th has long been considered to be an unlucky day, although many of the reasons for this designation have been long forgotten. There are enough superstitions relating to Friday and to the number thirteen individually that their conjunction would be the understandable cause for alarm among the superstitious. The reputation of this unfortunate date was further tainted when King Philip IV of France rounded up the Knights Templar for arrest on Friday the 13th of October 1307.

The appearance of Friday the 13th during the year can sometimes seem as though it is a random occurrence. The examination of several years time yields no recognizable pattern, with Friday the 13th occurring either one, two or three times during each year. The mysterious behavior of Friday the 13th is due entirely to the calendar that produces this date - the Gregorian calendar. Before examining our own calendar, let us first consider some of the difficulties inherent in making any calendar.

Challenges in Reconciliation

Attempting to reconcile the three major divisions of time (day, month, and year) has caused calendar-makers endless frustration throughout the centuries. The lunar and solar cycles vary slightly over time; modern estimates as to their mean length in days are currently 29.53089 and 365.242190 respectively. Because the lunar and solar cycles are not synchronized, and include a fractional day in their periods, a civilization would often use a process known as intercalation in an attempt to keep their count of days in alignment with the lunar and solar cycles. Intercalation is the process of manipulating the regular calendar to keep it in step with the seasons by adding days or months (our Gregorian calendar uses the process of leap years as a form of intercalation). Many different forms of intercalation of varying complexity have been used by different cultures. The best form of intercalation is one that is simple but accurate, however, due to the irreconcilable nature of the three major divisions of time, it is difficult to devise a system of intercalation that is both simple and accurate.

In the Babylonian Metonic cycle, seven out of every nineteen years contained thirteen lunar months instead of the standard twelve. Other forms of intercalation that adjusted the calendar seven times every nineteen years were also used in ancient Greece, China and India. Ancient Egypt used a calendar comprised of twelve thirty-day months followed by five intercalary days representing the birthdays of Osiris, Isis, Horus, Nephthys, and Set. The Mayan calendar was extraordinarily complex. Its Calendar Round involved interlocking cycles of thirteen, twenty and 365 days. The least common multiple of these numbers is 18,980, which meant that the Calendar Round lasted about 52 years. There was also a calendrical period known as the Long Count, which lasted about 5,000 years.

The Imperfect Precursor to the Gregorian Calendar: The Julian Calendar

The control of a civilization's calendar holds great political power. The Roman calendar was often manipulated by various rulers and by the college of pontifices. The college of pontifices would sometimes lengthen a year to increase the term of office for favored consuls and senators or shorten a year to decrease their rivals' terms. Until 304 BC, the Roman calendar was kept secret, so that only priests and aristocrats would know beforehand the occurrence of holidays, festivals and, most importantly, the dates when judicial and official business could be conducted. This ended when a plebe named Cneius Flavius stole a copy of the calendar and posted it on a white tablet in the Roman Forum for all the people to see. After this, the Roman calendar was issued as a public document.

The precursor to our current Gregorian calendar was the Julian calendar, which was introduced during the reign of Julius Caesar in 45 BC. At the time, the Roman calendar had become so hopelessly confused that the calendar was three months ahead of the seasons. In order to accommodate this discrepancy, the year previous to the introduction of the Julian calendar (46 BC) was known as "*ultimus annus confusionis*," or the "last year of confusion" and lasted 445 days.

In creating the Julian calendar, Julius Caesar relied upon the advice of the Alexandrian astronomer Sosigenes. He approximated the solar year as 365.25 days, which was very close, but 11 minutes 14 seconds longer than the actual year. Based on this approximation, the Julian calendar used a leap year every four years as an intercalation to make up for the fraction of a day. At first the 11 minutes 14 seconds discrepancy was too small to make any difference. However, over the course of many centuries, the fact that too many days were being added to the calendar became more and more noticeable.

Recognizing the Deficiencies of the Julian Calendar

As early as 725, the venerable Bede noticed that the full moon was occurring sooner than its scheduled date. Throughout the Middle Ages, a number of scholars, including John of Sacrobosco, Robert Grosseteste, and Roger Bacon pointed out the errors in the Julian calendar and proposed reforms. The main concern for the Catholic church was not so much the fact that the equinox was falling earlier in the calendar year, but that Easter was being celebrated at the wrong time.[•] Papal councils discussed the subject repeatedly throughout the 14th, 15th, and 16th centuries, but no decisive action was taken until the time of Pope Gregory XIII, who succeeded Pius V in 1572. Soon after he became Pope, Gregory established a commission of leading mathematicians, astronomers and church officials to examine proposals for calendar reform. In 1580, the commission returned a report recommending the reforms of a physician and astronomer named Luigi Giglio (also known as Aluise Lilio or Aloisius Lilius), who had died in 1576. The reforms were accepted and a papal bull was issued on February 24, 1582 ordering Christians to adopt the new calendar.^u The main difference between the Julian calendar and the Gregorian calendar is that the leap years of the century years (1700, 1800, 1900) are dropped, except for years that are divisible by 400 (1600, 2000, 2400, etc) which remain leap years. This adjustment allows the Gregorian calendar to remain in alignment with the seasons for over three thousand years without adjustment.

[•] The determination of Easter is a complex process. Amazingly, it takes 5,750,000 years for all the dates of Easter to repeat in the same order.

^u Different countries adopted the calendar at different times. England and the colonies did not adopt the Gregorian calendar until 1752.

“So what does all this have to do with Friday the 13th?”

There are fourteen possible calendars in the Gregorian system. One for January 1st falling on each of the seven days of the week in a standard 365 day year and one for January 1st falling on each of the seven days during a leap year. However, because of the interaction between the seven-day week and the four-year leap year cycle, the calendar must go through a 28-year period before returning to its starting place.

The number 365 is congruent to 1 modulo 7. In other words, there is a remainder of 1 when 365 is divided by 7. This causes the calendar to shift forward by one day each year. For instance, if January 1st is on Sunday during a non-leap year, it will fall on a Monday the following year. Without leap years, each date on the calendar would move forward one day per year and so would fall on a particular day every seven years. However, every four years, a leap year of 366 days causes each date to be pushed ahead two days, as 366 is congruent to 2 modulo 7 [see Table 1]. This irregularity is what causes the 28-year cycle of calendars, 28 being the least common multiple of 4 and 7.

Now the occurrences of Friday the 13th begin to make more sense. The examination of several years time will produce no pattern as the sequence of calendars repeats only once every 28 years. However, this does not take into account the Gregorian adjustment to the leap years, for three out of every four century years are not leap years. If we go back to the last century year before 2000 that was a leap year, an interesting pattern will emerge. The year 1600 was a leap year and occurred 18 years after the first countries began to adopt the Gregorian calendar, which makes it a convenient reference point. So, beginning in 1600, the calendar would progress through three 28-year periods before coming upon an irregularity.

The year 1700 would normally be a leap year and would keep the 28-year period running smoothly. Instead, what happens is that in the middle of the 28-year period there

occurs the year 1700, which should be a leap year, but by the Gregorian adjustment is not. This causes the previous 12 calendars to repeat and so the pattern is not resumed until 12 years later, which causes the 28-year period to stretch to 40 years[see Table 2]. The last appearance of the calendar for 1700 was only six years before, however, this came just two years prior to a leap year, whereas the next leap year after 1700 is four years hence. For this reason, the calendar for 1700 takes on the same role as the calendar for 1688. Although January 1, 1688 was a Thursday (not a Friday as in 1700), the fact that 1688 and 1700 both end on a Friday with the next leap year four years hence causes them to play similar roles in the calendrical procession.

The beauty of the Gregorian system lies not in its simplicity but rather in its symmetry. The calendar periods between 1600 and 2000 may best be considered as follows:

28 28 28 40 28 28 40 28 28 40 28 28 28

The first 40 year period contains 1700, the second contains 1800 and the third contains 1900. These ten periods of 28 years and three periods of 40 years make exactly 400 years, which is the overall period of the Gregorian calendar. A new 400-year cycle began last year, with 2000 corresponding to year 1 of the first 28-year cycle [see Table 1]. This means that 2000 began on a Saturday and had one Friday the 13th. The year 2001 corresponds to year 2 and has two Friday the 13ths and so on. The next year with three Friday the 13ths will be 2009.

There are many different ways to measure and keep track of time, and many different civilizations have devised ingenious methods to reconcile the lunar and solar cycles. All of these methods involve interesting mathematics, and some of them involve mathematical concepts that are quite complex. The study of calendars and the astronomical observations that serve as their basis continue to fascinate mankind from both a historical

and a modern standpoint. The complexity of the interactions that mark the passage of time have never ceased to confound and inspire human civilizations.

The cycle of calendars within the 28-year periods is the same, but because the century year occurs at progressively later points in the 40-year period, these cycles are somewhat different. In the charts below, each year is designated by the day on which January 1st occurs and whether or not the year is a leap year.

TABLE 1

<u>Year</u>	<u>Friday the 13ths</u>
1 SatLY	1
2 Mon	2
3 Tue	2
4 Wed	1
5 ThuLY	2
6 Sat	1
7 Sun	2
8 Mon	2
9 TuesLY	1
10 Thu	3
11 Fri	1
12 Sat	1
13 SunLY	3
14 Tue	2
15 Wed	1
16 Thu	3
17 FriLY	1
18 Sun	2
19 Mon	2
20 Tue	2
21 WedLY	2
22 Fri	1
23 Sat	1
24 Sun	2
25 MonLY	2
26 Wed	1
27 Thu	3
28 Fri	1

TABLE 2

First 40-year cycle		Second 40-year cycle		Third 40-year cycle		
Year	Friday the 13ths	Year	Friday the 13ths	Year	Friday the 13ths	
1	SatLY	1	1	1	SatLY	1
2	Mon	2	2	2	Mon	2
3	Tue	2	2	3	Tue	2
4	Wed	1	1	4	Wed	1
5	ThuLY	2	2	5	ThuLY	2
6	Sat	1	1	6	Sat	1
7	Sun	2	2	7	Sun	2
8	Mon	2	2	8	Mon	2
9	TueLY	1	1	9	TueLY	1
10	Thu	3	3	10	Thu	3
11	Fri	1	1	11	Fri	1
12	Sat	1	1	12	Sat	1
13	SunLY	3	3	13	SunLY	3
14	Tue	2	2	14	Tue	2
15	Wed	1	1	15	Wed	1
16	Thu	3	3	16	Thu	3
17	Fri(1700)	1	1	17	FriLY	1
18	Sat	1	1	18	Sun	2
19	Sun	2	2	19	Mon	2
20	Mon	2	2	20	Tue	2
21	TueLY	1	1	21	Wed(1800)	2
22	Thu	3	3	22	Thu	3
23	Fri	1	1	23	Fri	1
24	Sat	1	1	24	Sat	1
25	SunLY	3	3	25	SunLY	3
26	Tue	2	2	26	Tue	2
27	Wed	1	1	27	Wed	1
28	Thu	3	3	28	Thu	3
29	FriLY	1	1	29	FriLY	1
30	Sun	2	2	30	Sun	2
31	Mon	2	2	31	Mon	2
32	Tue	2	2	32	Tue	2
33	WedLY	2	2	33	WedLY	2
34	Fri	1	1	34	Fri	1
35	Sat	1	1	35	Sat	1
36	Sun	2	2	36	Sun	2
37	MonLY	2	2	37	MonLY	2
38	Wed	1	1	38	Wed	1
39	Thu	3	3	39	Thu	3
40	Fri	1	1	40	Fri	1

The two main sources for this paper were David Ewing Duncan's *Calendar: Humanity's Epic Struggle to Determine a True and Accurate Year* and E.G. Richards' *Mapping Time - The Calendar and its History*. Each of these books presents an enormous amount of information on historical astronomy, the calculation of the lunar and solar cycles, and the calendars of many different civilizations. In addition, they follow quite closely the development of the Roman, Julian and Gregorian calendars.

Jeremy Rifkin's *Time Wars* and Arno Borst's *The Ordering of Time* are excellent resources on the idea of time as a cultural construct. Rifkin's work explores in some detail how the system for ordering time adopted by a culture will greatly influence an individual's perception of time.

In the consideration of the higher mathematics involved in our current calendar, V. Frederick Rickey's "Mathematics of the Gregorian Calendar," in Issue #1 of the *Mathematics Intelligencer* for 1985 and Bernard Rasof's "Continued Fractions and 'Leap' Years," in *The Mathematics Teacher* of January 1970 each present some interesting considerations of optimal methods of intercalation. Duncan's and Richards' books also contain a certain amount of mathematics. Yury Grabovsky at Temple University has a few web pages that give a good introduction to the calendar and continued fractions as well.

This can be found at "<http://witt.math.temple.edu/~yury/calendar>".

References

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